Measurement of electrical power

Instantaneous, mean, active, reactive and apparent electrical power, power factor, etc.
We would like to remind you about these basic parameters in electronics and about three-phase measurement methods.

Definition of electrical power

At a given moment, when a current **i** travels from generator **G** to receiver **R** in the direction defined by the voltage **v** delivered by the generator *(figure 1)*, the **instantaneous power** supplied to the receiver R is equal to **product v·i.**

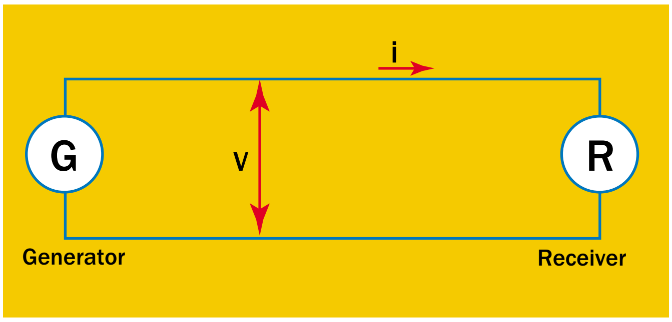

figure 1

If the voltage and current are DC, the mean power V·I is equal to the instantaneous power v·i.
If the voltage and current are **sinusoidal AC**, there is generally a **phase shift** φ between the voltage and the current *(figure 2)*.
The **instantaneous values** of voltage **v** and current **i** have the form:
$v = V_{max} \cos \omega t$
$i = I_{max} \cos (\omega t - \varphi)$
Where ω, the pulse, is proportional to the frequency F ($\omega = 2\pi F$).

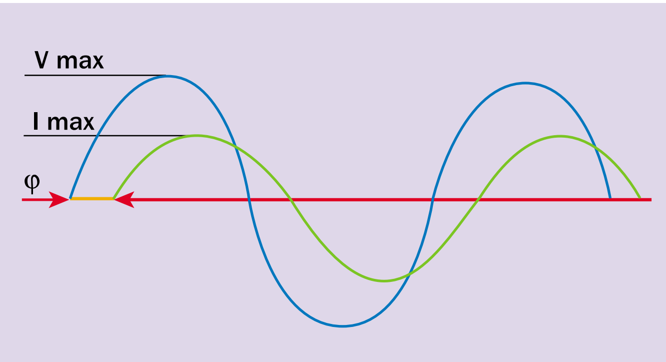

figure 2

The **phase shift** φ is, conventionally, counted as positive when the current is delayed in relation to the voltage.

The **instantaneous power** has a value of: $V_{max} \cdot I_{max} \cdot \cos \omega \cdot \cos (\omega t - \varphi)$.
You must take the average value of this product during a period to obtain the expression of the power provided by generator G to receiver R. This power is called the **active power** and is expressed by the formula:

$$P = \frac{V_{max} \cdot I_{max}}{\sqrt{2}} \cos \varphi = V_{eff} \cdot I_{eff} \cdot \cos \varphi$$

The **wattmeters** provide the expression of this product, either by causing a deviation of the pointer in the case of a device with an electrodynamic or ferrodynamic moving coil, or by supplying a DC current or a voltage proportional to the product in the case of electronic wattmeters; this current or this voltage is then applied to an analogue or digital display.

The existence of a phase shift φ between the current and the voltage leads, for AC currents, to the introduction of 3 additional quantities:

- **The apparent power $S = V_{eff} \cdot I_{eff}$,** in VA (volt-amperes), defining the voltage V_{eff} not to be exceeded (insulator breakdown, increase in core loss) and the intensity I_{eff} circulating in the receivers.

- **The power factor:**
  $$\cos \varphi = \frac{P}{S} = \frac{P}{V_{eff} \cdot I_{eff}}$$
  when the current and voltage are sinusoidal quantities.

- **The reactive power $Q = V_{eff} \cdot I_{eff} \cdot \sin \varphi$,** in rva (reactive volt-amperes). The latter may be directly measured by a wattmeter if for voltage $V_{max} \cdot \cos \omega t$ we substitute a phase-shifted voltage of $\pi/2$, i.e. $V_{max} \times \cos (\omega t - \pi/2)$.

The mean product measured will be

$V_{max} \cdot I_{max} \cdot \cos (\omega t - \pi/2) \times \cos (\omega t - \varphi)$ which is expressed by:

$$Q = \frac{V_{max} \cdot I_{max}}{\sqrt{2}} \cos (\pi/2 - \varphi) = V_{eff} \cdot I_{eff} \cdot \sin \varphi$$

Knowing P and Q, we can calculate the apparent power and the power factor:

Apparent power: $\mathbf{S = \sqrt{P^2+Q^2}}$

Power factor: $\mathbf{PF = P/S = P/\sqrt{P^2+Q^2}}$

Knowing the parameters defined above: active power, reactive power, apparent power, power factor, is fundamental in electrical engineering and enables accurate calculation of the characteristics of the equipment used: yield, load, cos φ, utilisation limits. The wattmeters used for these measurements are classified in three major families: electrodynamic, ferrodynamic and electronic.

Measurement of active power

4-wire balanced three-phase measurement (3 phases + neutral)

The intensities circulating in the three phases are equal in terms of rms values $I_1 = I_2 = I_3$ and show the same phase shift φ in relation to the respective voltages of the 3 phases.

If U_{1N} is the simple voltage measured between phase 1 and neutral, power P_1 supplied by phase 1 will be obtained by connecting a wattmeter as shown in figure 3.

Its value will be: $\mathbf{P_1 = U_{1N} \cdot I_1 \cdot \cos \varphi}$
The total power supplied P will be equal to 3 P_1.

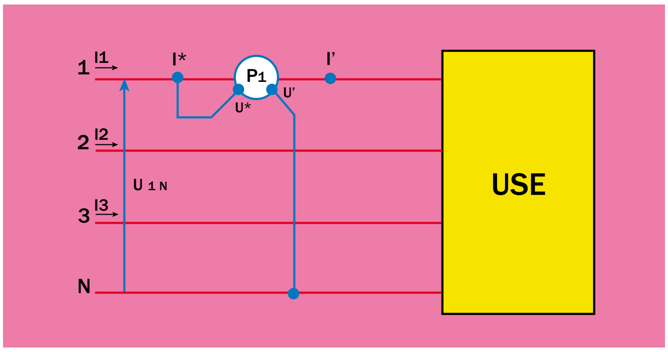

figure 3

Note: The expression $P_1 = U_{1N} \cdot I_1 \cdot \cos\varphi$ in the scalar product of the 2 vectors $\vec{U}_{1N}$ and $\vec{I}_1$ which enables use of the notation

$P_1 = \vec{U}_{1N} \cdot \vec{I}_1$

and in three-phase:

$P = \vec{U}_{1N} \cdot \vec{I}_1 + \vec{U}_{2N} \cdot \vec{I}_2 + \vec{U}_{3N} \cdot \vec{I}_3$

Measurement in 3-wire balanced three-phase (3 phases no neutral)

The intensities circulating in the three phases are equal $I_1 = I_2 = I_3$.
An artificial neutral is created using three resistors R, R et R'. The sum R' + r must be equal to R (r is the resistance of the voltage circuit of the unit).

This returns us to the previous case with U_{1N} between phase 1 and the artificial neutral *(figure 4)*.

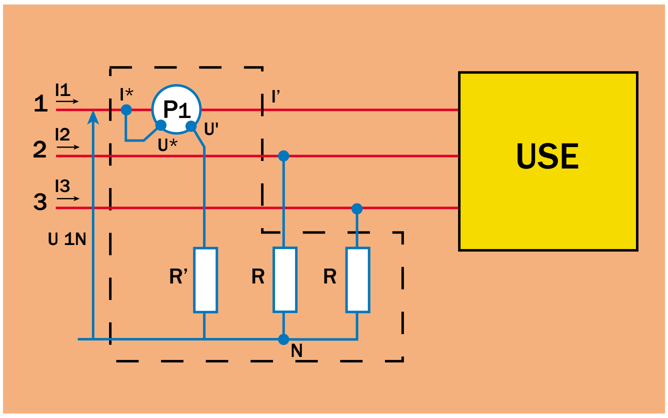

figure 4

P_1 = Power supplied on phase 1

Totale P supplied = $3\, U_{1N} \cdot I_1 \cdot \cos\varphi = 3P_1$.

With many wattmeters, the balanced three-phase measurements (3 phases no neutral) are performed directly; the artificial neutral point recreated by the resistors R, R and R' is included in the instrument (astatic wattmeter, CdA 778 wattmeter, for example). This design is shown in the diagram by the dotted section.

Measurement in 3-wire unbalanced three-phase (3 phases no neutral) - method using two wattmeters.

Whether the circuit is **balanced** or **not in the absence of a neutral, there remains $I_1 + I_2 + I_3 = 0$.**

In this case, the general expression of the power given above is simplified

$$P = (\vec{U}_{1N} - \vec{U}_{3N}) \cdot \vec{I}_1 + (\vec{U}_{2N} - \vec{U}_{3N}) \cdot \vec{I}_2$$

$$\text{so } P = \vec{U}_{13} \cdot \vec{I}_1 + \vec{U}_{23} \cdot \vec{I}_2$$

and the measurement of the total power may be carried out using two wattmeters *(figure 5)*.

U_{13} and U_{23} are the phase-to-phase voltages measured respectively between phase 1 and phase 3 and then between phase 2 and phase 3.

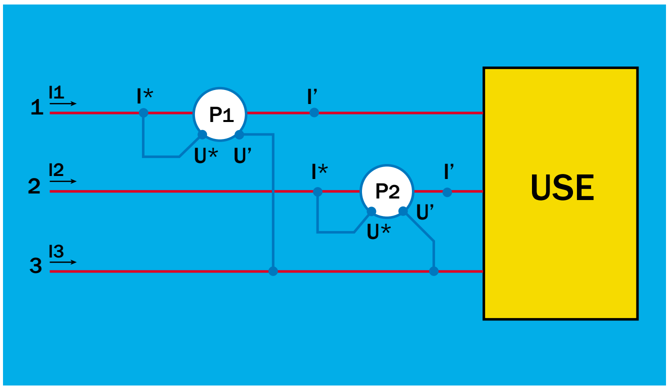

figure 5

Two cases may arise:

a) $P_1 \geq 0$ and $P_2 \geq 0$, then $P_{total} = P_1 + P_2$

b) one wattmeter deviates to the right and the other is as far as it will go to the left. To read the second; transfer the feed wires to the voltage circuit: U*-U' becomes U'-U*.
The value will be considered negative and we will obtain: $P_{total} = P_1 - P_2$

If it is a digital wattmeter we will add together the algebraic values displayed.

Note: it is possible to use a single wattmeter successively connected to 2 positions, using an inverter switch. This type of switch contains auxiliary contacts ensuring short-circuiting of the unused contacts.

Measurement in 4-wire balanced three-phase (3 phases + neutral)

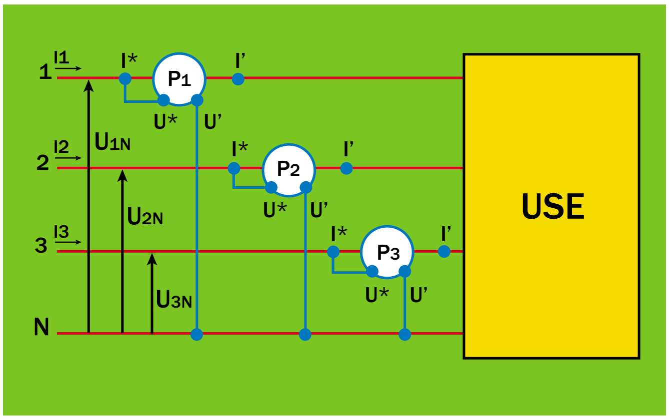

figure 6

We obtain $P_{total} = P_1 + P_2 + P_3$ *(figure 6)*.

In this case, we must use 3 wattmeters and add the readings together. If the measurement is stable, we can successively carry out 3 measurements with a single wattmeter. Caution: it is recommended to use a system preventing the intensity circuits from being cut off during switching.